

Online Learning of Portfolio Ensembles with Sector Exposure Regularization

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Abstract

We consider online learning of ensembles of portfolio selection algorithms and aim to regularize risk by encouraging diversification with respect to a predefined risk-driven grouping of stocks. Our procedure uses online convex optimization to control capital allocation to underlying investment algorithms while encouraging non-sparsity over the given grouping. We prove a logarithmic regret for this procedure with respect to the best-in-hindsight ensemble. We applied the procedure with known mean-reversion portfolio selection algorithms using the standard GICS industry sector grouping. Empirical Experimental results showed an impressive percentage increase of risk-adjusted return (Sharpe ratio).

1 Introduction

Online *portfolio selection* [Cover, 1991] has become one of the focal points in online learning research. So far, papers along this line have mainly considered *cumulative wealth* or *return* as the primary quantity to be optimized. However, the quality of any investment should rather be quantified using both return and *risk*, which often is measured by the variance of the return.¹ One of the major open challenges in online portfolio selection is the incorporation of effective mechanisms to dynamically control *risk* [Li and Hoi, 2014]. Within a *regret minimization* framework, like the one we consider here, this challenge is highlighted by the impossibility of achieving sub-linear regret in the adversarial setting with respect to risk adjusted measures such as the Sharpe Ratio [Even-Dar *et al.*, 2006]. This theoretical limitation is perhaps the main reason for the scarcity of papers studying risk control in the context of online portfolio selection.²

Following Johnson and Banerjee [2015] and motivated by the classic idea of diversifying risk factors and in particular, industry sectors (e.g., Grinold *et al.* [1989],

¹Many other notions of risk have been considered as well; see, e.g. Harlow [1991].

²This lack of risk-aware results in online (adversarial) portfolio learning stands in stark contrast to the situation in classic finance where the research into risk is vast within the context of portfolio allocation under distributional assumptions.

Cavaglia *et al.* [2000]), in this paper we present a portfolio selection ensemble learning procedure that applies any set of portfolio selection algorithms, and controls risk using sector *regularization*. The procedure invests in the stock market by dynamically allocating capital among the base algorithms, which provide their investment recommendations. These dynamic allocations are online learned using Newton steps, and risk is controlled using an ℓ_∞/ℓ_1 regularization term that penalizes over-exposed portfolios, where exposure is defined and quantified using a prior grouping of the stocks into “industry sectors.” While this regularization elicits diversification among groups, it also encourages a focus on the best convex combination of base algorithms within groups. The set of base trading algorithms can be arbitrary, but a typical application might be to activate within groups several independent copies of the same base algorithm instantiated with different values of its hyper-parameters, thus solving or alleviating the challenge of hyper-parameter tuning.

We prove a logarithmic regret bound for our procedure with respect to the best-in-hindsight ensemble of the base algorithms. We also show preliminary, promising numerical examples over a commonly used and challenging benchmark dataset. These results demonstrate impressive improvements in risk-adjusted return (Sharpe ratio) relative to direct applications of the base algorithms, and compared to a previous attempt to utilize group diversification.

1.1 Online Portfolio Selection

In Cover’s classic portfolio selection setting [Cover, 1991] (see also Borodin and El-Yaniv [1998], Chapt. 14), one assumes a market of n stocks. We consider an online game played through T days. On each day t the market is represented by a *market vector* \mathbf{X}_t of relative prices, $\mathbf{X}_t \triangleq (x_1^t, x_2^t, \dots, x_n^t)$, where for each $i = 1, \dots, n$, $x_i^t \geq 0$ is the *relative price* of stock i , defined to be the ratio of its closing price on day t relative to its closing price on day $t - 1$. We denote by $\mathbf{X} \triangleq \mathbf{X}_1, \dots, \mathbf{X}_T$ the sequence of T market vectors for the entire game. A *wealth allocation vector* or *portfolio* for day t is $\mathbf{b}_t \triangleq (b_1^t, b_2^t, \dots, b_n^t)$, where $b_i^t \geq 0$ is the wealth allocation for stock i . We require that the portfolio satisfy $\sum_{i=1}^n b_i^t = 1$. Thus, \mathbf{b}_t specifies the online player’s wealth allocation for each of the n stocks on day t , and b_i^t is the fraction of total current wealth invested in stock i on that day. We denote by $\mathbf{B} \triangleq \mathbf{b}_1, \dots, \mathbf{b}_T$ the sequence of T portfolios for the entire game. At the start of each trading day t , the player chooses a portfolio \mathbf{b}_t . Thus, by the end of day t , the player’s wealth is multiplied by $\langle \mathbf{b}_t, \mathbf{X}_t \rangle = \sum_{i=1}^n b_i^t x_i^t$, and assuming initial wealth of \$1, the player’s cumulative wealth by the end of the game is therefore

$$R_T(\mathbf{B}, \mathbf{X}) \triangleq \prod_{t=1}^T \langle \mathbf{b}_t, \mathbf{X}_t \rangle. \quad (1)$$

In the setting above, it is common to consider the logarithmic cumulative wealth, $\log R_T(\mathbf{B}, \mathbf{X})$, which can be expressed as a summation of the logarithmic daily wealth increases, $\log(\langle \mathbf{b}_t, \mathbf{X}_t \rangle)$.

The basic portfolio selection problem as defined above abstracts away various practical considerations, such as commissions, slippage, and more generally, market im-

pact, which are crucial for realistic implementations. While we do ignore these elements here as well, we focus in the present work on *risk*, and adapt the traditional definition of risk as the variance of return. Let R_A be the (annualized) return of an investment algorithm A and let σ_A be the (annualized) standard deviation of A 's return. Let R_f be the risk-free (annualized) interest rate. Then the (annualized) Sharpe ratio [Sharpe, 1966] of A is

$$S = \frac{R_A - R_f}{\sigma_A}.$$

For comparative purposes it is common to ignore the risk-free return R_f as we do here. The Sharpe ratio is thus a measure of risk-adjusted return, which captures the expected differential return per unit of risk.

In the online (worst-case) approach to portfolio learning the goal is to online generate a sequence $\{\mathbf{b}_t\}$ of portfolios that compete with the best-in-hindsight fixed portfolio, denoted \mathbf{b}_* . Denoting the round t loss of portfolio \mathbf{b} by $f_t(\mathbf{b})$ (in our case, $f_t(\mathbf{b}) = -\log(\langle \mathbf{b}_t, \mathbf{X}_t \rangle)$), we define the regret of sequence $\{\mathbf{b}_t\}$ as

$$\mathbf{Regret} \triangleq \sum_{t=1}^T (f_t(\mathbf{b}_t) - f_t(\mathbf{b}_*)).$$

In this paper we are mainly concerned with portfolio *ensembles*, where the allocation \mathbf{b}_t is over trading algorithms and \mathbf{b}_* represents the optimal-in-hindsight fixed allocation.

1.2 Risk Reduction by Sector Regularization

Diversification is the process of allocating wealth among investment choices such that exposure to certain “risk factors” is controlled or reduced. Markowitz’s modern portfolio theory (MPT) put diversification on front stage by showing a systematic diversification procedure for static portfolios using correlation analysis [Markowitz, 1952]. Understanding financial risk factors, and their interrelationship with stock returns, has been a longstanding challenge: one of the profound insights has been that risk factors can be manifested in many ways, but not all factors can be diversified [Sharpe, 1964]. Among the well-known diversifiable factors are those which are country specific and industry related [Grinold *et al.*, 1989]. Several decades ago diversification across countries provided greater risk reduction than industry-wise diversification [Solnik, 1995], but with increasing economic globalization, industry sector diversification has been found to be of increasing importance to active portfolio management [Cavaglia *et al.*, 2000].

The procedure proposed in this paper can, in principle, handle many types of diversification expressed in terms of predefined groupings of the stocks. While computation of effective groupings with sufficient predictive power is an interesting topic in and of itself, here we treat the grouping itself as available *prior knowledge*. Thus, for concrete validation of the proposed procedure, we focus on industry sector diversification, and our numerical examples make use of the *global industry classification standard* (GICS) – an industry taxonomy developed in 1999 by MSCI and Standard & Poor (S&P) for use by the global financial community.³

³The GICS structure consists of 10 sectors, 24 industry groups, 67 industries and 156 sub-industries.

1.3 Related Work and Contributions

We focus on online learning of sequential portfolios, and the main contextual anchor of the present work is the line of research pioneered by Cover [Cover, 1991; Cover and Ordentlich, 1996; Ordentlich and Cover, 1996], where the vanilla online portfolio selection problem was introduced and initially studied. Within this line the goal is to devise portfolio selection strategies whose cumulative wealth achieves, under adversarial inputs, sublinear regret (aka “universality”) with respect to an optimal in-hindsight “constant rebalanced portfolio” (CRP) strategy. The regret lower bound of $\Omega(\log T)$ [Ordentlich and Cover, 1996] for a T -round portfolio game was originally matched by Cover’s celebrated “Universal Portfolios” algorithm [Cover, 1991], and then rematched by various similar, or other “follow the leader” strategies (see, e.g., Helmbold *et al.* [1998], Blum and Kalai [1999], Agarwal *et al.* [2006]).

One of the approaches for regret minimization in the online portfolio setting is online convex optimization, where in each round t we consider a loss function, $f_t(\mathbf{b}_t) = -\log(\langle \mathbf{b}_t, \mathbf{X}_t \rangle)$, and exploit its convexity to ensure $O(\sqrt{T})$ regret from the best-in-hindsight CRP portfolio. Agarwal *et al.* [2006] showed that by using such a loss function that is exp-concave, it is possible to guarantee an improved regret bound of $O(\log T)$. This result was further extended by Das and Banerjee [2011], who proposed MA_{ons} to “ensemble learning,” whereby the portfolio learned is over algorithms rather than the stocks themselves. This resulted in $O(\log T)$ regret from the best-in-hindsight convex combination of algorithms.

To the best of our knowledge, there are only a few studies of risk control in the context of online portfolio learning with bounded regret. As mentioned earlier, Even-Dar *et al.* [2006] considered a more general expert setting and showed that one cannot achieve sub-linear regret with respect to the risk-adjusted return (Sharpe ratio). As a remedy, that paper considered optimizing global return in conjunction with a locally computed Sharpe ratio (defined over a recent historical window). Within a stochastic bandit setting, Shen *et al.* [2015] considered tracking the best expert with respect to the Sharpe ratio. The closest work to ours is that of Johnson and Banerjee [2015], who introduced the following group norm to encourage diversification. Consider a grouping $\mathcal{G} = (g_1, \dots, g_m)$ of the integers $[n] \triangleq \{1, \dots, n\}$, where $g_i \subseteq [n]$, $|g_i| = n_i$, and the groups g_i may overlap. For a vector $\mathbf{X} \in \mathbb{R}^n$, define its ℓ_∞/ℓ_1 group norm

$$L_{(\infty,1)}^{\mathcal{G}}(\mathbf{X}) = \|(\|\mathbf{X}_1\|_1, \dots, \|\mathbf{X}_m\|_1)\|_\infty, \quad (2)$$

where $\mathbf{X}_i \in \mathbb{R}^n$ equals \mathbf{X} , with all coordinates in $[n] \setminus g_i$ zeroed. This hierarchical group norm, which can be viewed as a kind of inverse to the group norm used in group Lasso [Yuan and Lin, 2006], encourages *non-sparsity* in its outer norm. Johnson and Banerjee [2015] presented an algorithm, called ORSAD, which uses this norm to encourage diversification in portfolio learning, thus leading to a reduction in several risk parameters, including a variant of the Sharpe ratio. The ORSAD algorithm consists of minimization steps of the form

$$\arg \min_{L_{(\infty,1)}^{\mathcal{G}}(\mathbf{b}_t) \leq K} -\eta \log(\langle \mathbf{b}_t, \mathbf{X}_t \rangle) + \frac{1}{2} \|\mathbf{b}_t - \mathbf{b}_{t-1}\|_2^2,$$

where K is a hyper-parameter. They also proved that their algorithm can guarantee $O(\sqrt{T})$ regret w.r.t. the best fixed portfolio in hindsight.

Although our procedure relies on the same ℓ_∞/ℓ_1 group norm to encourage diversification, our algorithm differs from that of Johnson and Banerjee in two ways: first, rather than generating portfolios on the stock themselves, we generate a weighted ensemble over investment algorithms. Second, our learning algorithm exploits the exp-concavity of our loss function, allowing the use of online Newton steps as in [Agarwal *et al.*, 2006] to guarantee $O(\log T)$ regret w.r.t. the best fixed ensemble in hindsight. A direct application of our procedure over the stocks themselves yields exponential improvement in regret relative to the result of Johnson and Banerjee [2015]. Also worth mentioning is that our implementation utilizes a fixed grouping of the stocks given by the standard GICS industry taxonomy, whereas Johnson and Banerjee [2015] employ a correlation-based heuristic to group the stocks on the fly. Our numerical examples in Section 3 include a direct comparison with the method of Johnson and Banerjee [2015], showing an overwhelming advantage to our method (see, e.g., Figure 1).

Algorithm 1 Ensemble Procedure (**EREP**)

- 1: Input: d trading algorithms, k groups, $T, \epsilon, \eta, \lambda > 0$.
- 2: Initialize: $\mathbf{P}_1, \mathbf{w}_1 = (\frac{1}{kd}, \dots, \frac{1}{kd}), A_0 = \epsilon I_{kd}$
- 3: **for** $t = 1$ to T **do**
- 4: **Play** \mathbf{w}_t and suffer loss $g_t(\mathbf{w}_t) + \lambda L_{(\infty,1)}^{\mathcal{G}}(\mathbf{w})$
- 5: **Receive** portfolios \mathbf{P}_{t+1} of base algorithms
- 6: **Update:** $A_t = A_{t-1} + \nabla g_t(\mathbf{w}_t)^T \nabla g_t(\mathbf{w}_t)$ and

$$\mathbf{w}_{t+1} = \arg \min_{\mathbf{w} \in \mathcal{B}} \{ \langle \nabla g_t(\mathbf{w}_t), \mathbf{w} - \mathbf{w}_t \rangle + \lambda L_{(\infty,1)}^{\mathcal{G}}(\mathbf{w}) + \eta \mathcal{D}_{A_t}(\mathbf{w} || \mathbf{w}_t) \}$$

7: **end for**

2 Exposure Regularized Ensemble Procedure

We consider a given grouping $\mathcal{G} = (g_1, \dots, g_k)$ of the integers $[n] \triangleq \{1, \dots, n\}$, where $g_i \subseteq [n]$, $|g_i| = n_i$. Each group g_i is called a *sector*. We assume that the sectors represent a meaningful structure of the n stocks and are not concerned here with how it is computed.

Our *exposure regularized ensemble procedure* (henceforth, **EREP**) is constructed over a set of d base-algorithms $\mathbf{A} = A_1, \dots, A_d$. For each $1 \leq i \leq d$, we create for algorithm A_i , k sub-algorithms $A_{i,j}$, $j = 1, \dots, k$, such that $A_{i,j}$ operates only over sector j . Therefore, **EREP** effectively operates over kd sub-algorithms. In each round t , **EREP** invests its current wealth in the sub-algorithms according to an allocation vector \mathbf{w}_t , which resides in the probability simplex. The actual allocation of wealth to individual stocks is calculated using \mathbf{w}_t , by aggregating the proposed portfolios by each of the sub-algorithms in a straightforward manner.

EREP requires the following definitions and notation. Let $A \in \mathbb{R}^{n \times n}$ be any positive-definite matrix. For $\mathbf{x}, \mathbf{w} \in \mathbb{R}^n$, the *Bregman divergence* generated by $F_A(\mathbf{w}) \triangleq \frac{1}{2} \mathbf{w}^T A \mathbf{w}$ is

$$\mathcal{D}_A(\mathbf{w}||\mathbf{x}) \triangleq \frac{1}{2} \|\mathbf{w} - \mathbf{x}\|_A^2 = \frac{1}{2} (\mathbf{w} - \mathbf{x})^T A (\mathbf{w} - \mathbf{x}).$$

We denote by I_n the unit matrix of order n . For a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, we denote by $\nabla f(\mathbf{w})$ its gradient (if it is differentiable) and by $f'(\mathbf{w})$ its subgradient.

We now explain the pseudo-code of **EREP** listed in Algorithm 1. In each round t , **EREP** first plays (rebalances its portfolio) according to already computed allocation vector \mathbf{w}_t (line 4). In response, the adversary selects a market vector (still line 4), which determines the following loss $g_t(\mathbf{w}_t)$:

$$g_t(\mathbf{w}) \triangleq -\log(\langle \mathbf{X}_t, \mathbf{P}_t \mathbf{w} \rangle), \quad (3)$$

where \mathbf{X}_t is the market vector selected by the adversary for round t . **EREP** then receives $\mathbf{P}_t \in \mathbb{R}^{n \times kd}$, the revised portfolios of its sub-algorithms (line 5). In line 6, **EREP** updates its allocation vector. In order to exploit the exp-concavity of the loss function, **EREP** utilizes the curvature of the loss function, as embedded in the matrix A_t , and then uses the Bregman divergence corresponding to A_t so as to optimize its allocation vector based on second order information (Newton step). The regularization term, $L_{(\infty, 1)}^G(\mathbf{w})$, encourages the simultaneous tracking of the most profitable sub-algorithm combination in each sector as well as diversification over sectors, as discussed in Sections 1.2 and 1.3.

2.1 Regret Analysis

In this section we analyze **EREP** and prove a logarithmic regret worst case bound. Let α be a positive real. A convex function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is α -exp-concave over the convex domain $\mathcal{B} \subset \mathbb{R}^n$ if the function $\exp(-\alpha f(\mathbf{x}))$ is concave. It is well known that the class of exp-concave functions strictly contains the class of strongly-convex functions. For example, the loss function typically used in online portfolio selection, $f_t(\mathbf{b}) = -\log(\langle \mathbf{b}, \mathbf{X}_t \rangle)$, is exp-concave but not strongly-convex.

The following two (known) basic lemmas concerning exp-concavity will be used in the proofs of Lemma 3 and Theorems 1 that follow.

Lemma 1. [Hazan et al., 2007] *Let f be an α -exp-concave over $\mathcal{B} \subset \mathbb{R}^n$ with diameter D , such that $\forall \mathbf{x} \in \mathcal{B}$, $\|\nabla f(\mathbf{x})\|_2 \leq G$. Then, for $\eta \leq \frac{1}{2} \min\{\alpha, \frac{1}{4GD}\}$, and for every $\mathbf{x}, \mathbf{y} \in \mathcal{B}$,*

$$f(\mathbf{y}) \geq f(\mathbf{x}) + \langle \nabla f(\mathbf{x}), \mathbf{y} - \mathbf{x} \rangle + \frac{\eta}{2} (\mathbf{y} - \mathbf{x})^T (\nabla f(\mathbf{x}) \nabla f(\mathbf{x})^T) (\mathbf{y} - \mathbf{x}).$$

Lemma 2. [Hazan et al., 2007] *Let $f_t : \mathbb{R}^n \rightarrow \mathbb{R}$ be α -exp-concave, and let A_t be as in Algorithm 1. Then, for $\eta = \frac{1}{2} \min\{\alpha, \frac{1}{4GD}\}$ and $\epsilon_0 = \frac{1}{\eta^2 D^2}$,*

$$\sum_{t=1}^T \|\nabla f_t(\mathbf{w}_t)\|_{A_t^{-1}}^2 \leq n \log T.$$

We consider a standard online convex optimization game [Zinkevich, 2003] where in each round t the online player selects a point \mathbf{w}_t in a convex set \mathcal{B} ; then a convex payoff function f_t is revealed, and the player suffers loss $f_t(\mathbf{w}_t)$. In an adversarial setting, where f_t is selected in the worst possible way, it is impossible to guarantee absolute online performance. Instead, the objective of the online player is to achieve sublinear regret relative to the best choice in hindsight, $\mathbf{w}_* \triangleq \arg \min_{\mathbf{w} \in \mathcal{B}} \sum_t f_t(\mathbf{w})$, where regret is

$$\mathbf{Regret} \triangleq \sum_{t=1}^T (f_t(\mathbf{w}_t) - f_t(\mathbf{w}_*)).$$

The mirror descent algorithm [Nemirovsky and Yudin, 1985; Beck and Teboulle, 2003] for online convex optimization was extended by Duchi *et al.* [2010] as follows. Instead of solving in each round

$$\mathbf{w}_{t+1} = \arg \min_{\mathbf{w} \in \mathbf{K}} \{ \eta \langle \nabla f_t(\mathbf{w}_t), \mathbf{w} - \mathbf{w}_t \rangle + \mathcal{D}(\mathbf{w} \| \mathbf{w}_t) \},$$

where $\mathcal{D}(x \| y)$ is the Bregman divergence generated by some strongly convex function ψ , they proposed to solve

$$\begin{aligned} \mathbf{w}_{t+1} = \arg \min_{\mathbf{w} \in \mathbf{K}} \{ & \eta \langle \nabla f_t(\mathbf{w}_t), \mathbf{w} - \mathbf{w}_t \rangle + \eta r(\mathbf{w}) \\ & + \mathcal{D}(\mathbf{w} \| \mathbf{w}_t) \}, \end{aligned}$$

where $r(\cdot)$ is some convex function which is not necessarily smooth. They proved that their revised method guarantees $O(\sqrt{T})$ regret relative to the best choice in hindsight whenever f is convex. Moreover, a sharper $O(\log(T))$ regret bound was shown for *strongly convex* f . This extension, called composite objective mirror descent (COMID), opened the door to applications in many fields and, in particular, to the possibility of using an ℓ_∞/ℓ_1 group norm as we do here. Our analysis of **EREP** thus boils down to extending the COMID framework for exp-concave functions.

We state without proof the following results (Lemma 3 and Theorem 1). Full proofs of these statements will be presented in the long version of this paper.

Lemma 3. *Let f_t be α -exp-concave over $\mathcal{B} \subset \mathbb{R}^n$ with diameter D , such that $\forall \mathbf{w} \in \mathcal{B}$, $\|\nabla f(\mathbf{w})\|_2 \leq G$. If \mathbf{w}_t is the prediction of Algorithm 1 in round t , then, for $\eta = \frac{1}{2} \min\{\alpha, \frac{1}{4GD}\}$ and for any $\mathbf{w}_* \in \mathcal{B}$,*

$$\frac{1}{\eta} [f_t(\mathbf{w}_t) - f_t(\mathbf{w}_*) + r(\mathbf{w}_{t+1}) - r(\mathbf{w}_*)] \leq$$

$$\mathcal{D}_{A_{t-1}}(\mathbf{w}_* \| \mathbf{w}_t) - \mathcal{D}_{A_t}(\mathbf{w}_* \| \mathbf{w}_{t+1}) + \frac{1}{2\eta^2} \|\nabla f_t(\mathbf{w}_t)\|_{A_t^{-1}}^2.$$

Theorem 1. *Let f_t be α -exp-concave over $K \subset \mathbb{R}^n$, $\eta = \frac{1}{2} \min\{\alpha, \frac{1}{4GD}\}$ and let $\epsilon_0 = \frac{1}{\eta^2 D^2}$. If $(\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_T)$ are the predictions of Algorithm 1, then for any fixed point $\mathbf{w}_* \in \mathcal{B}$,*

$$\sum_{t=1}^T (f_t(\mathbf{w}_t) + r(\mathbf{w}_t) - f_t(\mathbf{w}_*) - r(\mathbf{w}_*)) = O(\log T).$$

Corollary 1. For Algorithm 1, for appropriate⁴ $\epsilon, \eta > 0$, and every $\lambda \geq 0$, for any fixed point $\mathbf{w}_* \in \mathcal{B}$, it holds that

$$\begin{aligned} \sum_{t=1}^T g_t(\mathbf{w}_t) + \lambda L_{(\infty,1)}^{\mathcal{G}}(\mathbf{w}) - g_t(\mathbf{w}_*) - \lambda L_{(\infty,1)}^{\mathcal{G}}(\mathbf{w}_*) \\ = O(\log T). \end{aligned}$$

3 Numerical Examples

In this section we present a preliminary empirical study examining and analyzing the performance of **EREP** on the well-known SP500 benchmark dataset [Borodin *et al.*, 2000]. This challenging dataset consists of 25 stocks over a period of 5 years, from 1998 to 2003, which includes the dot-com crash. Qualitatively similar results to those presented here will be presented for other datasets in the extended version of this paper.

The stocks in the SP500 dataset were categorized into the following 4 sectors according to the global industry classification standard (see, e.g., Yahoo Finance): Technology, Finance, Healthcare, and Services. Fixing this sector grouping throughout our study we examine, in our first experiment, how well **EREP** controls and operates sets of base algorithms. We selected the following set of base algorithms, all of which are implemented in the Li *et al.* OLPS simulation library [Li *et al.*, 2015]:

- Exponentiated Gradient (EG) [Helmbold *et al.*, 1998]: this classic algorithm is among the early universal algorithms. EG is typically not a strong contender in empirical studies and we include it as a control point, to verify that our strategy avoids using its portfolios.
- Anticor [Borodin *et al.*, 2004]: one of the first algorithms designed to aggressively exploit mean-reversion via (anti) correlation analysis.
- Online Moving Average Reversion (OLMAR) [Li and Hoi, 2012]: designed to exploit mean-reversion based on moving average predictions. OLMAR is known to be a strong performer in many benchmark datasets.

We examined two different settings for the base algorithms. In the first setting (called “Mixed”), the set of base algorithms consists of the three algorithms: EG, Anticor and OLMAR.⁵ In the second setting (called “Olmarm only”), we took three instances of OLMAR applied with the following values of its window size (a critical hyper-parameter): 10, 15, 20.

The critical hyper-parameter of **EREP** is λ , which controls the ℓ_∞/ℓ_1 regularization intensity. Preliminary empirical measurements of the dynamic range of the average maximal sector weight as a function of λ showed that $\lambda = 0.1$ roughly corresponds to the median of this range. Therefore, to obtain a rough impression we initially applied

⁴In our applications we used the same parameters that were used in Agarwal *et al.* [2006]. In general, these parameters can be calibrated according to market variability; see Agarwal *et al.* [2006].

⁵All hyper-parameters of the base algorithms were set to the recommended default parameters in the OLPS simulator. EG (with $\eta = 0.05$), Anticor (with $w = 20$) and OLMAR (with $w = 20, \epsilon = 10$).

EREP with this setting. In a more elaborate experiment we optimized λ using a standard walk-forward procedure [Pardo, 1992]; see details below.

Table 1: SP500 Dataset: Sharpe ratio performance of **EREP** and benchmark algorithms

Setting	Base Algorithms			MA _{ons}	ORSAD	EREP	
						$\lambda = 0.1$	λ_{WF}
Mixed	EG	Anticor _(w=20)	OLMAR _(w=20)	0.91	0.52	1.12	1.14
	0.51	0.90	0.94				
Olmar only	$w = 10$	$w = 15$	$w = 20$	0.97	0.52	1.33	1.39
	0.90	0.77	0.94				

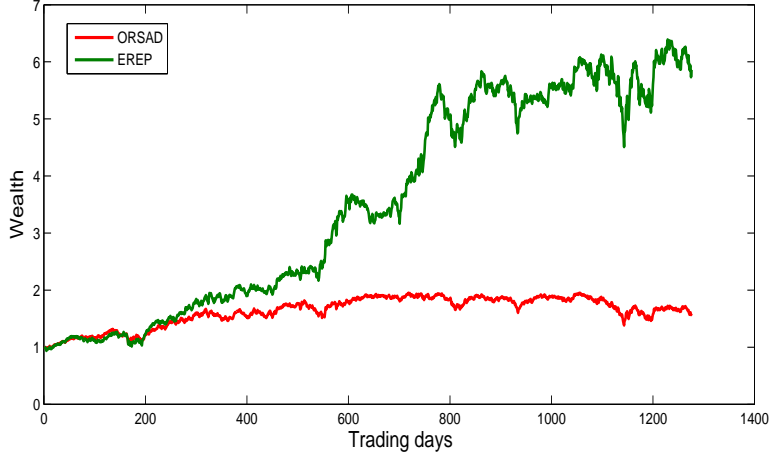


Figure 1: EREP compared to ORSAD

Table 1 summarizes the Sharpe ratio results obtained for these runs of **EREP** compared to various benchmark algorithms. Consider first the Mixed setting (first row of the table). Here we see that **EREP** improves the Sharpe ratio of the best algorithm by almost 20%. In the Olmar only setting, the Sharpe ratio improvement relative to the best Olmar instance is by over 40%. Similar improvements can be seen with respect to the MA_{ons} ensemble procedure of Das and Banerjee [2011], applied here on the same sets of base algorithms. Turning now to the ORSAD algorithm of Johnson and Banerjee [2015], which uses the same ℓ_∞/ℓ_1 regularization (but with a different learning algorithm applied over the stocks), we see an improvement of over 100% in the Sharpe ratio. Furthermore, in Figure 1 we see the cumulative return curves of both **EREP** and ORSAD for the entire 5-year period, showing that the return itself has also improved by hundreds of percents.

While these results clearly provide a compelling proof of concept for the effective-

ness of **EREP**, we chose $\lambda = 0.1$ in hindsight, which affects both the return and the Sharpe ratio of the algorithm. Is it possible to calibrate λ online? To this end we employed a standard walk-forward procedure whereby λ was sequentially optimized over a sliding window of size w periods so as to improve the Sharpe ratio. Although this routine eliminates the need to choose λ , it introduces w as a new hyper-parameter. Figure 2 depicts the sensitivity of the overall Sharpe ratio with respect to the window size $w \in [10, 300]$. It is evident that this procedure is not sensitive to choices of w in this range. **EREP**'s improvement of the Sharpe ratio for the two base-algorithms settings is shown in Table 1 (under λ_{WF}). The cumulative return of **EREP** with sequentially calibrated λ appears in Figure 1.

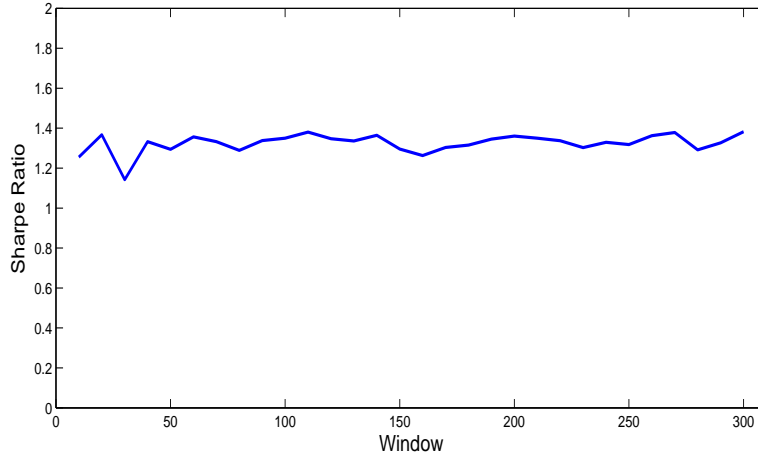


Figure 2: Sharp ratio sensitivity w.r.t w

4 Conclusions

In this paper we show how to effectively use the ℓ_∞/ℓ_1 regularization to improve risk-adjusted return performance. While the original work introducing this norm [Johnson and Banerjee, 2015] considered portfolios on the stocks themselves, we propose to incorporate this structured norm within a Newton style optimization and apply the learning algorithm over trading strategies based on a known partition of the stocks into industry sectors. Along the way, we also online optimize the choice of hyper-parameters and/or the choice of underlying trading algorithms. Our preliminary empirical study indicates that the proposed procedure can achieve a substantial improvement in the Sharpe ratio relative to the base algorithms themselves, relative to their ensemble using the technique of Das and Banerjee [2011]. Moreover, it substantially improves the original method of Johnson and Banerjee [2015]. Further empirical evidence with qualitatively similar

conclusions has been established and will be reported in the long version of this paper.

Johnson and Banerjee [2015] attempted to dynamically compute sector groupings based on correlations. This idea is very attractive in the absence of prior knowledge or in fast changing markets, and can be further extended to explore hierarchical structures in the stock market as discussed, e.g., in Mantegna [1999]. However, the regret guarantee in Johnson and Banerjee [2015] cannot hold in its current form using a dynamically changing grouping of the stocks. It would be very interesting to extend their learning algorithm or ours to capture dynamically changing groupings. While the proposed procedure allows for improved Sharpe-ratios, it is still a “long-only” strategy, which does not utilize short selling. While attractive in certain regulated settings like mutual funds, it cannot achieve market neutrality. It would be very interesting to extend our technique so as to be market neutral. As a final caveat, we must emphasize that we have focused on an idealized “frictionless” setting that excludes various elements such as commissions, slippage and market impact. While this setting is a reasonable starting point for considering risk reduction, these elements must be considered in real-life applications.

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